Government of **Western Australia**
School Curriculum and Standards Authority

WACE
Western Australian Certificate of Education

Western Australian Certificate of Education Examination, 2016

Question/Answer Booklet

MATHEMATICS SPECIALIST

Please place your student identification label in this box

Section One: Calculator-free

Student Number: In figures | | | | | | | | |

In words _____

_____

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

2013/42079

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	54	35
Section Two: Calculator-assumed	14	14	100	102	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Western Australian external examinations are detailed in the *Year 12 Information Handbook 2016*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

Section One: Calculator-free **35% (54 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

Question 1 **(5 marks)**

Find all the values, real and complex, of x for which $H(x) = 0$ if

$$H(x) = -5x^3 + 25x^2 - 20x + 100.$$

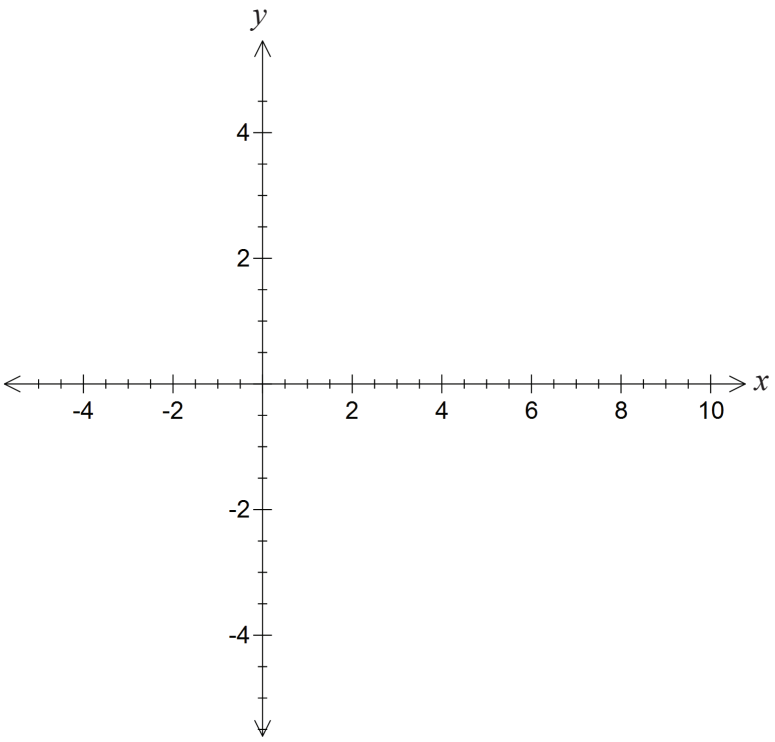
Question 2 **(8 marks)**

The functions f and g are given by

$$f(x) = 3 - \sqrt{x} \text{ and } g(x) = (3-x)^2 .$$

(a) Show that the function defined by $y = f(g(x))$ is defined for all real values of x. (3 marks)

(b) On the axes below, sketch the composite function $y = f(g(x))$. (2 marks)



(c) How should the domain of $g(x)$ be changed so that $f(x)$ and $g(x)$ are inverse functions of each other? (3 marks)

Question 3 **(9 marks)**

(a) Let y represent the income of a small nation, a the amount of the income spent on necessities and b the percentage of the remaining income spent on luxuries. The economic model that relates these three quantities is

$\frac{dy}{dt} = k(1-b)(y-a)$, where t is the time in years.

Given that b is 65%, express y in terms of a, k and t, where k is a constant. (4 marks)

(b) Solve the differential equation

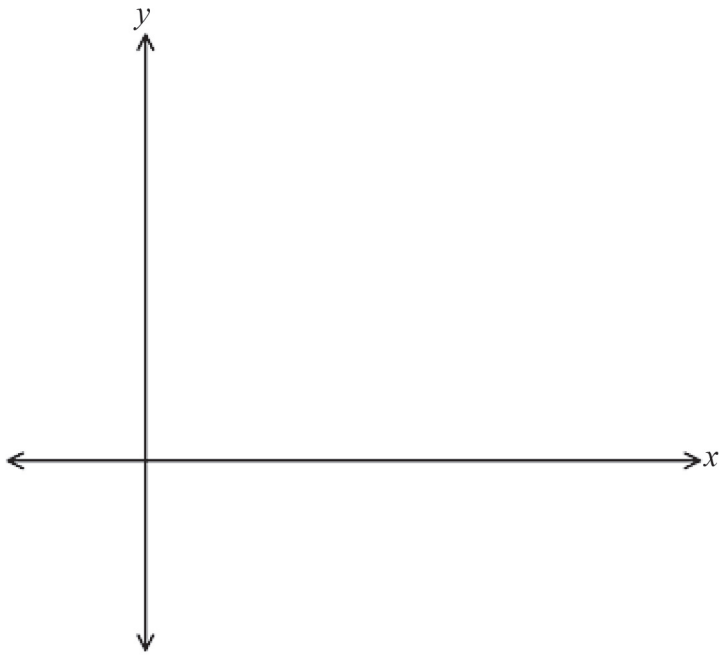
$$xe^{x^2} + yy' = 0$$

subject to the initial condition that $y = 1$ when $x = 0$. (5 marks)

Question 4 **(9 marks)**

The function $f(x)$ is defined for $x > 0$ by $f(x) = \dfrac{-2 + 3x - x^2}{x^2 - x}$.

(a) Sketch the graph of $f(x)$ on the axes below. (4 marks)



(b) What is the range of $f(x)$? (1 mark)

(c) Show that $f^{-1}(x) = \dfrac{2}{x+1}$, $x > -1$, $x \neq 1$, and state the domain of $f^{-1}(x)$. (2 marks)

(d) Sketch the graph of $f^{-1}(x)$ on the same axes used for part (a). Label your sketch clearly. (2 marks)

See next page

Question 5 **(8 marks)**

(a) Evaluate $V = \int_0^2 2\pi x(x^2 + 2)dx$ using substitution and express your answer in exact form. (4 marks)

(b) Determine $\int (\sin^2 4x \cos 4x)\, dx$. (4 marks)

See next page

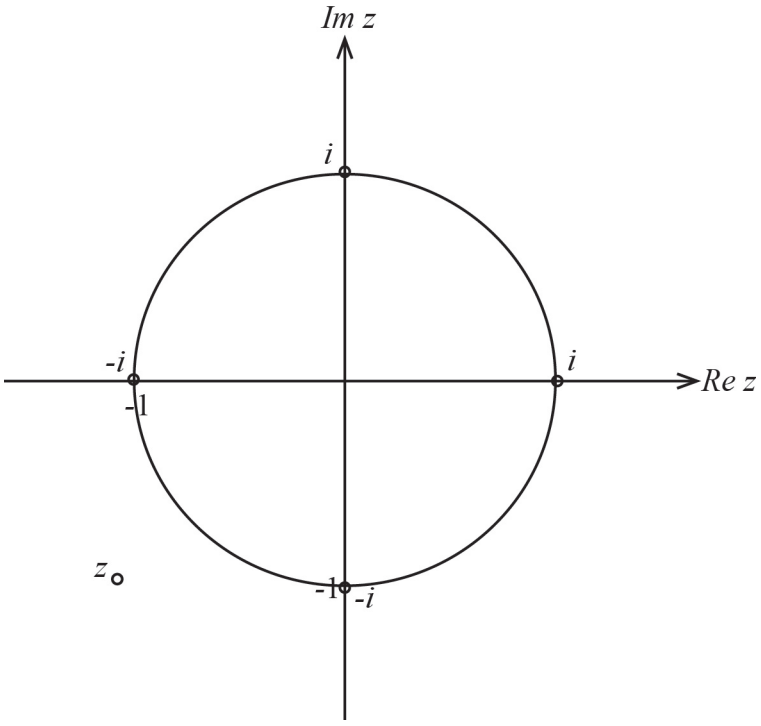
Question 6 **(7 marks)**

(a) Given z is a complex number with modulus r and argument θ, express the modulus and argument of each of the complex numbers z_1 and z_2 in terms of r and θ where

(i) $z_1 = \bar{z}$. (2 marks)

(ii) $z_2 = -z^{-1}$. (3 marks)

(b) The diagram below shows the circle in the complex plane and the position of the complex number z.

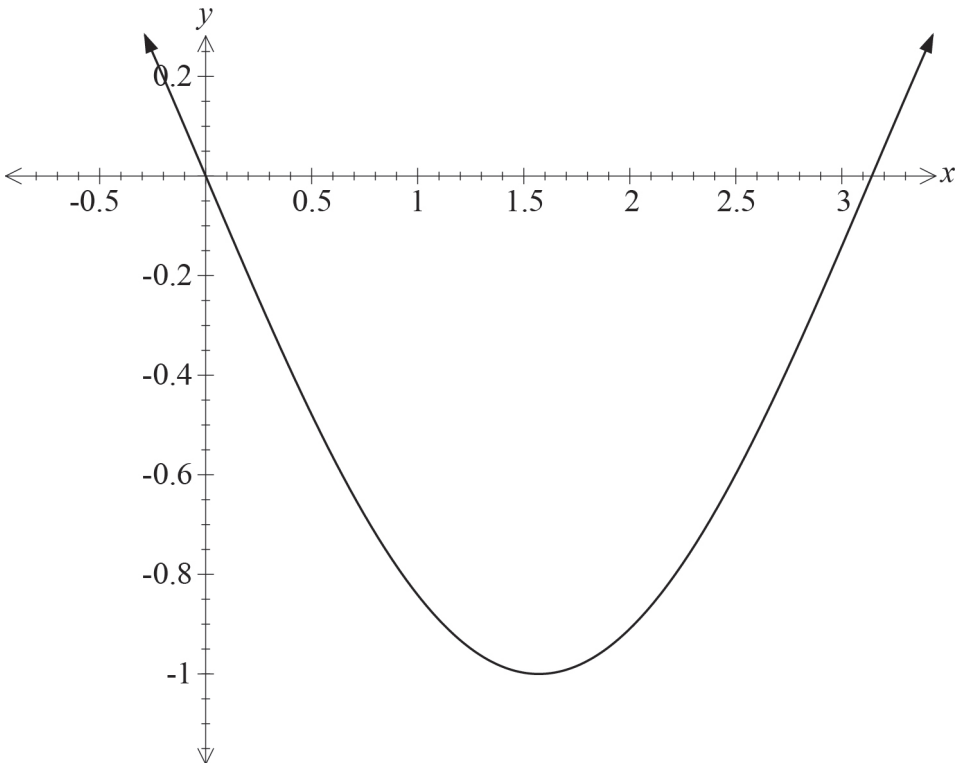


Given the approximate values for r and θ are 1.5 and 220° respectively, indicate the locations of the complex numbers z_1 and z_2 as defined in part (a) on the diagram above. (2 marks)

See next page

Question 7 **(8 marks)**

A region is bounded by the x-axis and one arc of the graph of $y = -\sin x$ as shown in the diagram below.



(a) Evaluate $\int_0^{\pi} (-\sin x)\, dx$. (3 marks)

(b) Given that the curve is defined for all values of x, evaluate $\int_{-7\pi}^{8\pi} (-\sin x)\, dx$. (1 mark)

(c) Calculate the exact volume of the solid generated by rotating the region shown in the diagram around the x-axis. (4 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

Published by the School Curriculum and Standards Authority of Western Australia
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OSBORNE PARK WA 6017